

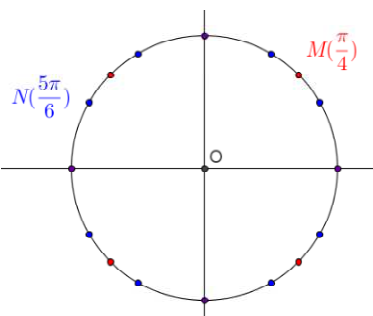
Ex1 : la mesure principale de $\frac{5\pi}{3}$ est $-\frac{\pi}{3}$. $\cos \frac{5\pi}{3} = \frac{1}{2}$; $\sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

la mesure principale de $\frac{-5\pi}{4}$ est $\frac{3\pi}{4}$. $\cos \left(\frac{-5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$; $\sin \left(\frac{-5\pi}{4}\right) = \frac{\sqrt{2}}{2}$

la mesure principale de $\frac{-11\pi}{2}$ est $\frac{\pi}{2}$. $\cos \left(\frac{-11\pi}{2}\right) = 0$; $\sin \left(\frac{-11\pi}{2}\right) = 1$

la mesure principale de $\frac{-41\pi}{6}$ est $-\frac{5\pi}{6}$. $\cos \left(\frac{-41\pi}{6}\right) = -\frac{\sqrt{3}}{2}$; $\sin \left(\frac{-41\pi}{6}\right) = -\frac{1}{2}$

Ex2 :



$(\vec{OM}, \vec{ON}) = \frac{5\pi}{6} - \frac{\pi}{4} (2\pi) = \frac{7\pi}{12} (2\pi)$
 et de $(\vec{ON}, \vec{OM}) = -\frac{7\pi}{12} (2\pi)$

Ex3 : $(\vec{u}, \vec{v}) = \frac{-\pi}{7} (2\pi)$ et $(\vec{u}, \vec{w}) = \frac{-\pi}{4} (2\pi)$.

$(\vec{v}, \vec{w}) = (\vec{v}, \vec{u}) + (\vec{u}, \vec{w}) (2\pi) = -(\vec{u}, \vec{v}) + \frac{-\pi}{4} (2\pi) = \frac{\pi}{7} - \frac{\pi}{4} (2\pi) = \frac{-3\pi}{28} (2\pi)$

$(\vec{w}, \vec{v}) = -(\vec{v}, \vec{w})(2\pi) = \frac{3\pi}{28} (2\pi)$

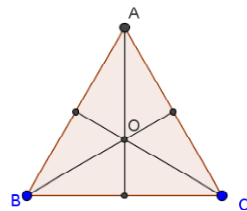
$(-\vec{w}, \vec{v}) = (\vec{w}, \vec{v}) + \pi(2\pi) = \frac{3\pi}{28} + \pi(2\pi) = \frac{31\pi}{28}(2\pi) = \frac{-25\pi}{28}(2\pi)$

$(2\vec{w}, \vec{u}) = (\vec{w}, \vec{u}) (2\pi) = \frac{\pi}{4} (2\pi)$

$(-\vec{u}, -\vec{v}) = (\vec{u}, \vec{v})(2\pi) = \frac{-\pi}{7} (2\pi)$

$(\vec{u}, -\vec{v}) = (\vec{u}, \vec{v}) + \pi(2\pi) = \frac{-\pi}{7} + \pi(2\pi) = \frac{6\pi}{7} (2\pi)$

Ex4 : $(\vec{BA}, \vec{AC}) = (\vec{-AB}, \vec{AC})(2\pi) = (\vec{AB}, \vec{AC}) + \pi (2\pi) = \frac{\pi}{3} + \pi (2\pi) = \frac{4\pi}{3} (2\pi)$
 $= -\frac{2\pi}{3} (2\pi)$



$(\vec{OB}, \vec{OC}) = \frac{2\pi}{3} (2\pi)$

$(\vec{AB}, \vec{OC}) = (\vec{IB}, \vec{IC})(2\pi) = \frac{\pi}{2} (2\pi)$

$(\vec{CA}, \vec{AB}) = (\vec{-AC}, \vec{AB}) (2\pi) = (\vec{AC}, \vec{AB}) + \pi (2\pi)$
 $= \frac{-\pi}{3} + \pi (2\pi) = \frac{2\pi}{3} (2\pi)$

Ex5 : 1° Dans $[-\pi ; 0]$: $\cos a = -0,4$. Or $\cos^2 a + \sin^2 a = 1$, donc $(-0,4)^2 + \sin^2 a = 1$
 $\sin^2 a = 1 - 0,16 = 0,84$

Donc $\sin a = \sqrt{0,84}$ ou $\sin a = -\sqrt{0,84}$

à rejeter car $a \in [-\pi ; 0]$

Donc $\sin a = -\sqrt{0,84}$

2° Dans $[0 ; \pi]$: $4 \cos^2 x - 3 = 0$ donc $\cos^2 x = \frac{3}{4}$

donc $\cos x = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$ ou $\cos x = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$

donc dans $[0 ; \pi]$ $x = \frac{\pi}{6}$ $x = \frac{5\pi}{6}$

3° $\sin b = 0,73$

En radians, la calculatrice donne $b \approx 0,82$ rad.

A l'aide du cercle trigonométrique, on rajoute $b \approx \pi - 0,82$ rad $\approx 2,32$ rad.

Ex6 : Mesures principales de :

$(\vec{AC}, \vec{AE}) = \frac{\pi}{4} (2\pi)$

$(\vec{BC}, \vec{BF}) = \frac{\pi}{3} + \frac{\pi}{4} (2\pi) = \frac{7\pi}{12} (2\pi)$

$$(\overrightarrow{BD}, \overrightarrow{BF}) = \frac{\pi}{4} + \frac{7\pi}{12} (2\pi) = \frac{10\pi}{12} (2\pi) = \frac{5\pi}{6} (2\pi)$$

$$\begin{aligned} (\overrightarrow{BA}, \overrightarrow{AC}) &= (\overrightarrow{-AB}, \overrightarrow{AC}) (2\pi) = (\overrightarrow{AB}, \overrightarrow{AC}) + \pi (2\pi) = \frac{\pi}{3} + \pi (2\pi) = \frac{4\pi}{3} (2\pi) \\ &= -\frac{2\pi}{3} (2\pi) \end{aligned}$$

$$(\overrightarrow{DC}, \overrightarrow{CA}) = (\overrightarrow{-CD}, \overrightarrow{CA}) (2\pi) = (\overrightarrow{CD}, \overrightarrow{CA}) + \pi (2\pi) = \frac{-\pi}{4} - \frac{\pi}{3} + \pi (2\pi) = \frac{5\pi}{12} (2\pi)$$

$$\begin{aligned} (\overrightarrow{EA}, \overrightarrow{CB}) &= (\overrightarrow{EA}, \overrightarrow{EC}) + (\overrightarrow{EC}, \overrightarrow{CB}) (2\pi) = \frac{\pi}{3} + (\overrightarrow{CE}, \overrightarrow{CB}) + \pi (2\pi) \\ &= \frac{\pi}{2} + \frac{7\pi}{12} + \pi (2\pi) = \frac{25\pi}{12} (2\pi) = \frac{\pi}{12} (2\pi) \end{aligned}$$

Ex7 : Approfondissement :

a) Utilisons SOH CAH TOA : $\cos \hat{C} = \frac{1}{2}$ donc $\hat{C} = \frac{\pi}{2}$ donc $(\overrightarrow{CA}, \overrightarrow{CB}) = \frac{\pi}{3} (2\pi)$

b) $(\overrightarrow{AD}, \overrightarrow{AE}) = (\overrightarrow{AD}, \overrightarrow{AC}) + (\overrightarrow{AC}, \overrightarrow{AB}) + (\overrightarrow{AB}, \overrightarrow{AE}) (2\pi) = -\frac{\pi}{3} - \frac{\pi}{2} - \frac{\pi}{3} (2\pi)$
 $= -\frac{7\pi}{6} (2\pi) = \frac{5\pi}{6} (2\pi)$

Ex8 : Approfondissement :

On sait que le triangle ABC est isocèle en C, soit $x = \hat{A} = \hat{B}$, on a $\frac{\pi}{6} + 2x = \pi (2\pi)$

$$\text{donc } 2x = \pi - \frac{\pi}{6} (2\pi) = \frac{5\pi}{6} (2\pi)$$

$$\text{donc } x = \frac{5\pi}{12} (2\pi)$$

a) $(\overrightarrow{BJ}, \overrightarrow{CA}) = (\overrightarrow{BJ}, \overrightarrow{BA}) + (\overrightarrow{BA}, \overrightarrow{CA}) (2\pi) = \frac{\pi}{4} + (\overrightarrow{-AB}, \overrightarrow{-AC}) (2\pi)$
 $= \frac{\pi}{4} + (\overrightarrow{AB}, \overrightarrow{AC}) (2\pi) = \frac{\pi}{4} + \frac{5\pi}{12} (2\pi) = \frac{8\pi}{12} (2\pi) = \frac{2\pi}{3} (2\pi)$

b) $(\overrightarrow{JB}, \overrightarrow{BC}) = (\overrightarrow{-BJ}, \overrightarrow{BC}) (2\pi) = (\overrightarrow{BJ}, \overrightarrow{BC}) + \pi (2\pi)$
 $= -\left(\frac{5\pi}{12} - \frac{\pi}{4}\right) + \pi (2\pi) = \frac{10\pi}{12} (2\pi) = \frac{5\pi}{6} (2\pi)$